Q.8 Solve : $\frac{d^2y}{dx^2} + y = \sin x \sin 2x$

Sol. We have,

$$\frac{d^2y}{dx^2} + y = \sin x \sin 2x$$

$$(D^2+1)y = \sin x \sin 2x$$

A.E. is $m^2+1=0 \Rightarrow m = \pm i$

C.F. $= C_1 \cos x + C_2 \sin x$

P.I. $= \frac{1}{D^2+1}\sin x \sin 2x = \frac{1}{D^2+1}\frac{1}{2}[\cos x - \cos 3x]$

$$= \frac{1}{2}\left[\frac{1}{D^2+1}\cos x - \frac{1}{D^2+1}\cos 3x\right]$$

$$= \frac{1}{2}\left[x\frac{1}{2D}\cos x - \frac{1}{-9+1}\cos 3x\right] = \frac{1}{2}\left[\frac{x}{2}\sin x + \frac{1}{8}\cos 3x\right]$$

$$= \frac{1}{16}[4x\sin x + \cos 3x]$$

Complete Solution is $y =$ C.F.+P.I.

$\Rightarrow$ $y = C_1\cos x + C_2 \sin x + \frac{1}{16}(4x\sin x + \cos 3x)$

Q.9 Obtain the general solution of the differential equation

$$y'' - 2y' + 2y = x + e^x \cos x$$

Sol. We have, $y'' - 2y' + 2y = x + e^x \cos x$

A.E. is $m^2 - 2m + 2 = 0 \Rightarrow m = 1 \pm i$

$$C.F.=e^{x}(A\cos x+B\sin x)$$

$$P.I.=\frac{1}{D^2-2D+2}x+\frac{1}{D^2-2D+2}e^{x}\cos x$$

Where $$I_1=\frac{1}{D^2-2D+2}x=\frac{1}{2\left[1-D+\frac{D^2}{2}\right]}x=\frac{1}{2\left[1-\left(D-\frac{D^2}{2}\right)\right]}x$$

$$=\frac{1}{2}\left[1-\left(D-\frac{D^2}{2}\right)\right]^{-1}x=\frac{1}{2}\left[1+\left(D-\frac{D^2}{2}\right)+...\right]x$$

$$=\frac{1}{2}\left[x+Dx-\frac{D^2}{2}x+...\right]=\frac{1}{2}[x+1]$$

and $$I_2=\frac{1}{D^2-2D+2}e^{x}\cos x$$

$$=e^{x}\frac{1}{(D+1)^2-2(D+1)+2}\cos x=e^{x}\frac{1}{D^2+1}\cos x=e^{x}.x\frac{1}{2D}\cos x$$

$$=\frac{1}{2}xe^{x}\sin x\left[\text{If } f(-a^2)=0,\text{ then }\frac{1}{f(D^2)}\phi(x)=x\frac{1}{f'(D)}\phi(x)\right]$$

$$y=C.F.+P.I.$$

$$=e^{x}(A\cos x+B\sin x)+\frac{1}{2}(x+1)+\frac{1}{2}xe^{x}\sin x.$$ Ans.

Q.10 Solve : $(D^2-4D+4)y=x^3e^{2x}$

Sol. We have, $(D^2-4D+4)y=x^3e^{2x}$

A.E. is $m^2-4m+4=0 \quad \Rightarrow (m-2)^2=0 \quad \Rightarrow m=2,\ 2$

C.F. $=(C_1+C_2x)e^{2x}$

P.I. $$=\frac{1}{D^2-4D+4}x^3.e^{2x}=e^{2x}\frac{1}{(D+2)^2-4(D+2)+4}x^3$$

$$= e^{2x}\frac{1}{D^2}x^3 = e^{2x}.\frac{1}{D}\left(\frac{x^4}{4}\right) = e^{2x}.\frac{x^5}{20}$$

The complete solution is $y = (C_1 + C_2 x)e^{2x} + e^{2x}.\frac{x^5}{20}$ Ans

Q.11 Solve $(D^4 - 1)y = e^x \cos x$

Sol. Here, we have

$$(D^4 - 1)y = e^x \cos x$$

A.E. is $m^4 - 1 = 0 \Rightarrow (m+1)(m-1)(m^2+1) = 0$

$\Rightarrow$ $m = -1, 1, +i, -i$

C.F. $= C_1 e^x + C_2 e^x + (C_3 \cos x + C_4 \sin x)$

P.I. $= \frac{1}{D^4 - 1} e^x \cos x$

$$= e^x \frac{1}{(D+1)^4 - 1}\cos x = e^x \frac{1}{D^4 + 6D^3 + 4D^2 + 6D}\cos x$$

$$= e^x \frac{1}{(-1)^2 + 6(-1)D + 4(-1) + 6D}\cos x$$

$$= e^x \frac{1}{1 - 6D - 4 + 6D}\cos x = -\frac{e^x \cos x}{3}$$

Complete solution is $y = C.F. + P.I.$

$\Rightarrow y = C_1 e^{-x} + C_2 e^x + (C_3 \cos x + C_4 \sin x) - \frac{e^x \cos x}{3}$ Ans.