(6)

$$\frac{x^2}{a^2} + \frac{y^2}{a^2+\lambda} = 1 \quad ; \quad \lambda \text{ being a parameter} \rightarrow (1)$$

Soln.→ Diff. above w.r.t. 'x'

$$\frac{2x}{a^2} + \frac{2yy'}{a^2+\lambda} = 0 \quad \text{where } y' = \frac{dy}{dx}$$

i.e.

$$\frac{x}{a^2} = -\frac{yy'}{a^2+\lambda} \rightarrow (2)$$

from (1)

$$\frac{x^2}{a^2} - 1 = \frac{-y^2}{a^2+\lambda} \Rightarrow \frac{x^2-a^2}{a^2} = \frac{-y^2}{a^2+\lambda} \rightarrow (3)$$

Divide (2) & (3) we have

$$\frac{x}{x^2-a^2} = \frac{yy'}{y^2}$$

Now $y' = \frac{dy}{dx}$ is replaced by $-\frac{dx}{dy}$

i.e.

$$\frac{x}{x^2-a^2} = \frac{1}{y}\left(-\frac{dx}{dy}\right)$$

seprate the variables

$$y\,dy = -\frac{(x^2-a^2)}{x}\,dx$$

$$y\,dy = -x\,dx + a^2\frac{1}{x}\,dx$$

Integrating above eqn

$$\int y\,dy = -\int x\,dx + a^2\int \frac{1}{x}\,dx + \frac{c}{2}$$

$$\frac{y^2}{2} = -\frac{x^2}{2} + a^2\log x + \frac{c}{2}$$

$$\boxed{x^2 + y^2 - 2a^2\log x = c}$$

Ans

(11)

Soln :- Let $y = cx$ is the family of straight line

Diff. above eqn we have

$$y' = c = \text{constant}$$

Elimination c by above two equation i.e.

$$\left.\begin{aligned} y &= cx \\ y' &= c \end{aligned}\right\} \Rightarrow y' = \frac{y}{x}$$

Replace $y'$ with $\left(-\frac{1}{y'}\right)$ we have

$$-\frac{1}{y'} = \frac{y}{x}$$

$$y' = -\frac{x}{y}$$

we know that $y' = \frac{dy}{dx}$

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$y\,dy = -x\,dx$$

Integrating above eqn

$$\int y\,dy = -\int x\,dx.$$

$$\frac{y^2}{2} = -\frac{x^2}{2} + \frac{a^2}{2} \qquad \left\{\because \frac{a^2}{2} = \text{constant of integration}\right\}$$

$$\boxed{y^2 + x^2 = a^2}$$ Ans