

Th:  $\rightarrow$  If  $A$  be any  $n$ -rowed square matrix, then

$$(\text{Adj } A) A = A (\text{Adj } A) = |A| I_n$$

Proof:  $\rightarrow$  let  $A = [a_{ij}]_{n \times n}$  be any ~~non~~ square matrix

let  $\text{Adj } A = [b_{ij}]_{n \times n}$

Then  $b_{ij} = A_{ji} = \text{co-factor of } a_{ji} \text{ in } |A|$ .

Since the matrices  $A$  &  $\text{Adj } A$  are both  $n \times n$  square matrix, therefore both products  $A(\text{Adj } A)$  &  $(\text{Adj } A) \cdot A$  exist and also of  $n \times n$ .

Also, the  $(i, j)^{\text{th}}$  element of  $A(\text{Adj } A)$

$$= \sum_{k=1}^n a_{ik} b_{kj} \quad \left\{ \text{by def}^n \text{ of Product of two matrices} \right\}$$

$$= \sum_{k=1}^n a_{ik} A_{jk} \quad \left\{ \text{from } \textcircled{1} \right\}$$

$$= 0 \text{ or } |A| \text{ according as } i \neq j$$

Hence the  $(i, j)^{\text{th}}$  element of  $A(\text{Adj } A) = |A|$  if  $i = j$   
&  $= 0$  if  $i \neq j$ .

Therefore  $A(\text{Adj } A) = \begin{pmatrix} |A| & 0 & 0 & 0 & \dots & 0 \\ 0 & |A| & 0 & 0 & \dots & 0 \\ \vdots & & & & & \\ 0 & 0 & 0 & 0 & \dots & |A| \end{pmatrix}$

$$A (\text{Adj } A) = |A| I_n$$