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BCA Sem - 2

Computer Organization and Architecture

Topic

**BOOLEAN ALGEBRA**

Boolean algebra :- Boolean algebra is a algebraic system that is developed by handling the binary system . this algebra is proposed by George Boole, a English mathematician in the year 1854. This is an alternative form of Aristotle propositional logic using symbol 0 and 1 or True and false.

This is the algebra in which propositions are limits to either 0 or 1, known as Boolean number and expression of calculation form with this number is known as Boolean expression. Variable that store Boolean number is Boolean variables.

Boolean expression takes the form :-

$P + Q = R$  here  $P, Q, R$  is the Boolean variables that can store only two value 0, 1.

Followings are important Boolean operations

1. Boolean addition operation -> It is operation of logical addition represented

by (+) that work with two Boolean number/variable and produce on Boolean number as result.

$$0 + 0 = 0$$

$$1 + 0 = 1$$

$$0 + 1 = 1$$

$$1 + 1 = 1$$

2. Logical Multiplication operation  $\rightarrow$  ( . )  
this operation is represented with dot symbol and perform operation between two Boolean number and return result as third number. It is reverse of addition .

$$1.1 = 1$$

$$0.1 = 0$$

$$1.0 = 0$$

$$0.0 = 0$$

3. Logical Complement operation :- this operation work with single Boolean number or expression and represented by ( X ) or ( ' ) . it just reverse value of the Boolean number as  $\rightarrow$

$$0 = 1$$

$$1 = 0$$

Boolean expression generally execute from left to right by the following rules :

1. Scanning of expression generally done from left to right
2. If expression consist of () then expression within ().
3. Operator executed in the order , complement first, then multiplication, and at last addition .

$$Z=P+(\bar{Q}.R+S)$$

The basic rules of Boolean algebra are :

$$A + 0 = A$$

$$A + 1 = 1$$

$$A.0 = 0$$

$$A.1 = A$$

$$A + A = A$$

$$A.A = A$$

$$A+A' = 1$$

$$A.A' = 0$$

$$A'' = A$$

$$A+A.B = A$$

$$A+A'B = A+B$$

$$(A+B).(A+C) = A+B.C$$

Proofing of Basic postulates:

we use truth table

Truth table is the tabular representation of Boolean operation by giving the possible value as input of the operation and getting output result of the operation . it may be created as –

Rule –I column of the truth table is taken as operand and expression of the operation

Rule –II all possible values combinations are taken separately in row and their possible output has shown in their appropriate place.

Rule –III number of column and row depends upon the operand taken and possible basic operation of expression along with the total basic combination of binary value of the data

Truth table of the Basic Rule \_I  
( $(A+B).(A+C)=(A+BC)$ )

Truth Table

A	B	C	A+B	A+C	$(A+B).(A+C)$	BC	A+BC
0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	1	1	1	1
1	0	0	1	1	1	0	1
1	0	1	1	1	1	0	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

## BOOLEAN OPERATION:

1. The And Operation  $\rightarrow$  binary multiplication
2. The or operation  $\rightarrow$  binary addition
3. The not operation  $\rightarrow$  binary complement

Principle of duality :- it helps in deducing new theorem and statement without proving them. According to principle of duality, the swapping of 0 and 1, + and . in any statement of Boolean algebra does not change the tureness of the statement.

$$(A+1)+(A+BC) = 1 \rightarrow (A.0) (A.B+C) = 0$$

A	B	C	A+1	BC	A+BC	(A+1)+(A+BC)	
0	0	0	1	0	0	1	
0	0	1	1	0	0	1	
0	1	0	1	0	0	1	
0	1	1	1	1	1	1	
1	0	0	1	0	1	1	
1	0	1	1	0	1	1	

1	1	0	1	0	1	1	
1	1	1	1	1	1	1	

$$(A+1)+(A+BC) = 1 \rightarrow (A.0) (A.B+C) = 0$$

A	B	C	A0	AB	(A.B+C)	(A.0)(A.B+C)	
0	0	0	0	0	0	0	
0	0	1	0	0	1	0	
0	1	0	0	0	0	0	
0	1	1	0	0	1	0	
1	0	0	0	0	0	0	
1	0	1	0	0	1	0	
1	1	0	0	1	1	0	
1	1	1	0	1	1	0	

$$A(B+C) = (A.B)+(A.C) \rightarrow A+(BC) = (A+B).(A+C)$$

A	B	C	B+C	A.(B+C)	A.B	A.C	(AB)+(AC)	
0	0	0	0	0	0	0	0	
0	0	1	1	0	0	0	0	
0	1	0	1	0	0	0	0	
0	1	1	1	0	0	0	0	
1	0	0	0	0	0	0	0	
1	0	1	1	1	0	1	1	



1	1	0	1	1	1	0	1	
1	1	1	1	1	1	1	1	

$$A(B+C) = (A.B)+(A.C) \rightarrow A+(BC) = (A+B).(A+C)$$

A	B	C	A+C	A+B	BC	A+BC	(A+B).(A+C)	
0	0	0	0	0	0	0	0	
0	0	1	1	0	0	0	0	
0	1	0	0	1	0	0	0	
0	1	1	1	1	1	1	1	
1	0	0	1	1	0	1	1	
1	0	1	1	1	0	1	1	
1	1	0	1	1	0	1	1	
1	1	1	1	1	1	1	1	

## OTHER BASIC LAW OF BOOLEAN ALGEBRA

1. Law of multiplication :- If a Boolean variables multiplied by itself ANY number of times , the result will be the original Boolean variable

$$A.A.A.A=A$$

A	A.A	A.A.A	A.A.A.A	
0	0	0	0	
1	1	1	1	

2. Law of addition :- If a Boolean variables added by itself ANY number of times , the result will be the original Boolean variable

$$A+A+A+A=A$$

A	A+A	A+A+A	A+A+A+A	
0	0	0	0	
1	1	1	1	

3. COMMUTATIVE LAW -> The result of multiplication AND addition performed on any two Boolean variables will be the same irrespective of order of the variables .

$$A.B=B.A$$

$$A+B=B+A$$

A	B	B.A	A.B	A+B	B+A
0	0	0	0	0	0
0	1	0	0	1	1
1	0	0	0	1	1
1	1	1	1	1	1

4. Associative Law :- the resultant of multiplication and addition performed on two or more variables will be the same irrespective of the grouping of Boolean variables .

$$A.B.C .D = (A.B).(C.D) = A.(B.C.D) = (A.B).(C.D) = (A.B.C).D = \dots\dots$$

$$A+B+C+D = (A+B)+C = A+(B+C) \dots\dots\dots$$

5. Distributive Law :- law combination of multiplication and addition of Boolean variables

$$A(B+C) = A.B + A.C \quad \rightarrow \quad A+BC = (A+B).(A+C)$$