

(I). Symmetric Relation -

A relation R in A is called a symmetric relation if for every $(x, y) \in R$, $(y, x) \in R$

Example -

(i) Let $A = \{1, 2, 3\}$.

Then $R = \{(1, 2), (2, 1), (3, 3)\}$ is a relation in A .

Here,

$$(1, 2) \in R, (2, 1) \in R$$

$$(2, 1) \in R, (1, 2) \in R$$

$$(3, 3) \in R, (3, 3) \in R.$$

Thus for every $(x, y) \in R$, $(y, x) \in R$.

$\therefore R$ is a symmetric relation in A .

(ii) Let $A = \{1, 2, 3\}$

Then $R = \{(1, 1), (1, 3), (2, 3)\}$ is a relation in A .

Here,

$$(1, 1) \in R, (1, 1) \in R$$

$$(1, 3) \in R \text{ but } (3, 1) \notin R.$$

$\therefore R$ is not a symmetric relation in A .

(III). Transitive Relation -

A relation R in A is called a transitive relation if

for every $(x, y) \& (y, z) \in R$, $(x, z) \in R$.

Example -

Let $A = \{1, 2, 3\}$

Then $R = \{(1, 2), (2, 3), (1, 3)\}$ is a relation in A .

Here,

$(1,2) \& (2,3) \in R, (1,3) \in R.$
 Thus for every $(x,y) \& (y,z) \in R, (x,z) \in R.$
 $\therefore R$ is a transitive relation in $A.$

(ii) Let $A = \{1,2,3\}$

Then $R = \{(1,1), (1,2), (2,3)\}$
 is a relation in $A.$

Here,

$(1,1) \& (1,2) \in R, (1,2) \in R.$
 $(1,2) \& (2,3) \in R, \text{ but } (1,3) \notin R$
 $\therefore R$ is not a transitive relation
 in $A.$

(IV) Equivalence Relation —

A relation R in A is called
 an equivalence relation if

(i) R is a reflexive relation in A
 i.e., for every $x \in A, (x,x) \in R$

(ii) R is a symmetric relation in A
 i.e., for every $(x,y) \in R, (y,x) \in R$ and

(iii) R is a transitive relation in A
 i.e., for every $(x,y) \& (y,z) \in R, (x,z) \in R.$