Department of BBA, R.N.College, Hajipur
Dr.M.K.Singh

DUAL LINEAR PROGRAMMING PROBLEMS

For every linear programming problem there is a corresponding linear programming problem called the dual. If the original problem is a maximization problem then the dual problem is minimization problem and if the original problem is a minimization problem then the dual problem is maximization problem. In either case the final table of the dual problem will contain both the solution to the dual problem and the solution to the original problem.

The solution of the dual problem is readily obtained from the original problem solution if the simplex method is used. The formulation of the dual problem also sometimes referred as the concept of duality which is helpful for the understanding of the linear programming. The variable of the dual problem is known as the dual variables or shadow price of the various resources. The dual problem is easier to solve than the original problem. The dual problem solution leads to the solution of the original problem and thus efficient computational techniques can be developed through the concept of duality. Finally, in the competitive strategy problem solution of both the original and dual problem is necessary to understand the complete problem.

Dual Problem Formulation

If the original problem is in the standard form then the dual problem can be formulated using the following rules:

- The number of constraints in the original problem is equal to the number of dual variables. The number of constraints in the dual problem is equal to the number of variables in the original problem.
- The original problem profit coefficients appear on the right hand side of the dual problem constraints.
- If the original problem is a maximization problem then the dual problem is a minimization problem. Similarly, if the original problem is a minimization problem then the dual problem is a maximization problem.
- The original problem has less than or equal to (≤) type of constraints while the dual problem has greater than or equal to (≥) type constraints.
- The coefficients of the constraints of the original problem which appear from left to right are placed from top to bottom in the constraints of the dual problem and vice- versa.

The Dual Linear Programming Problem is explained with the help of the following Example:

The primal problem is given in the following form:

Max. $Z = c_1x_1+c_2x_2+\ldots\ldots\ldots+c_nx_n$

subject to $a_{11}x_1+ a_{12}x_2 +\ldots\ldots+a_{1n}x_n \le b_1$

$a_{21}x_1 + a_{22}x_2 +\ldots\ldots+a_{2n}x_n \le b_2$

$\vdots \quad \vdots \quad \vdots \quad \vdots$

$\vdots \quad \vdots \quad \vdots \quad \vdots$

$a_{m1}x_1 + a_{m2}x_2 +\ldots\ldots+a_{mn}x_n\le b_m$

and $x_1,x_2,\ldots\ldots x_n \ge 0$

The dual of this problem is expressed as:

Min. $Z^* = b_1y_1 + b_2y_2 + \ldots\ldots\ldots\ldots + b_ny_n$

Subject to $a_{11}y_1 + a_{21}y_2 + \ldots\ldots\ldots + a_{m1}y_m \geq c_1$

$a_{12}y_1 + a_{22}y_2 + \ldots\ldots\ldots\ldots + a_{m2}y_m \geq c_2$

: : : :

: : : :

$a_{1n}y_1 + a_{2n}y_2 + \ldots\ldots\ldots\ldots + a_{mn}y_m \geq c_n$

and $y_1, y_2, \ldots\ldots, y_m \geq 0$

where $y_1, y_2, \ldots\ldots y_m$ are dual decision variables.

Note : The dual of the dual is primal.

Numerical example:

Max. $Z = 50\ x_1 + 120\ x_2$

Subject to $2\ x_1 + 4\ x_2 \leq 80$

$3\ x_1 + x_2 \leq 60$

Where $x_1, x_2 \geq 0$

The dual of this primal is :

Min $Z^* = 80\ y_1 + 60\ y_2$

Subject to $2\ y_1 + 3\ y_2 \geq 50$

$4\ y_1 + y_2 \geq 60$ where $y_1, y_2 \geq 0$

Assignments

Write down the dual of the given primal and find the optimal solution.

1. Min. $Z = 40x_1 + 200x_2$

Subject to $4x_1 + 40x_2 \geq 160$

$3x_1 + 10x_2 \geq 60$

$8x_1 + 10x_2 \geq 80$

$x_1, x_2 \geq 0$

2. Min. $Z = x_1 + x_2$

Subject to $2x_1 + x_2 \geq 4$

$x_1 + 7x_2 \geq 7$

$x_1, x_2 \geq 0$

3. Min. $Z = 40x_1 + 200x_2$

4. Subject to $4x_1 + 40x_2 \geq 160$

$3x_1 + 10x_2 \geq 60$

$8x_1 + 10x_2 \geq 80$

$x_1, x_2 \geq 0$