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Establish the Thin Lens Formula $\frac{1}{v}-\frac{1}{u}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ by using Fermat's principle.

Fermat's principle can be used to establish the thin lens formula $\frac{1}{v}-\frac{1}{u}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ without using the law of refraction.

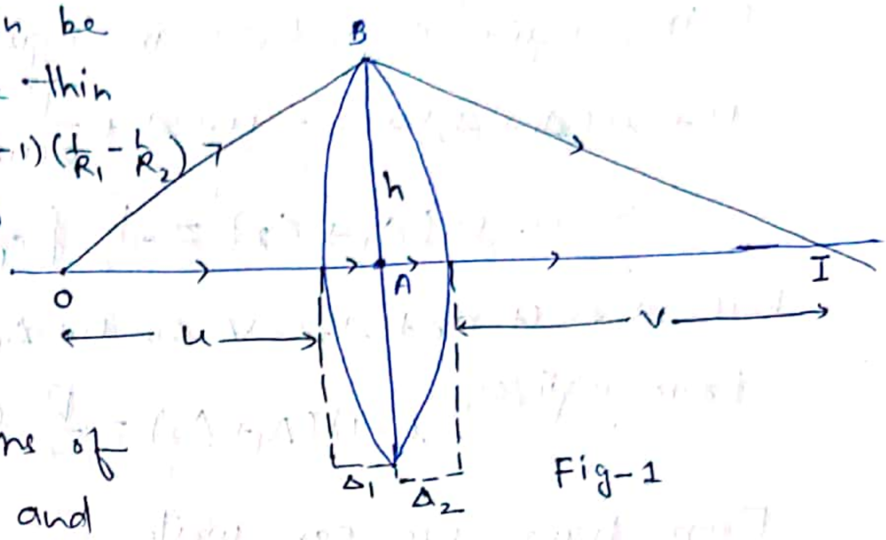

Fig-1

We consider two paths through the lens of refractive index μ and radii of curvatures R_1 and R_2. One a straight line path OAI connecting object point O and image point I and other path OBI by touching the edge B of the lens as shown in fig-1.

Time taken by the light to cover the path OAI is

$$t_1=\{u+\mu.(\Delta_1+\Delta_2)+v\}/c \quad \text{——— (1)}$$

Time taken by the light to cover the path OBI is

$$t_2=\left\{\sqrt{(u+\Delta_1)^2+h^2}+\sqrt{(v+\Delta_2)^2+h^2}\right\}/c \quad \text{——— (2)}$$

Using Fermat's principle of stationary time,

$$t_1=t_2$$

$$\Rightarrow \{u+\mu(\Delta_1+\Delta_2)+v\}/c=\left\{\sqrt{(u+\Delta_1)^2+h^2}+\sqrt{(v+\Delta_2)^2+h^2}\right\}/c$$

$$\Rightarrow u+\mu(\Delta_1+\Delta_2)+v=\sqrt{(u+\Delta_1)^2+h^2}+\sqrt{(v+\Delta_2)^2+h^2} \quad \text{——— (3)}$$

For paraxial approximation, $h \ll u+\Delta_1$ and $h \ll v+\Delta_2$

Now $\sqrt{(u+\Delta_1)^2+h^2}=(u+\Delta_1)\left[1+\left(\frac{h}{u+\Delta_1}\right)^2\right]^{\frac{1}{2}}$

$$=(u+\Delta_1)\left[1+\frac{1}{2}\left(\frac{h}{u+\Delta_1}\right)^2\right]$$ By using Binomial theorem.

$$\sqrt{(u+\Delta_1)^2+h^2} = u+\Delta_1+\frac{h^2}{2(u+\Delta_1)} \quad \text{——— (4)}$$

Similarly $$\sqrt{(v+\Delta_2)^2+h^2} = v+\Delta_2+\frac{h^2}{2(v+\Delta_2)} \quad \text{——— (5)}$$

Using eqns (4) and (5) in eqn (3), we get

$$u+\mu(\Delta_1+\Delta_2)+v = u+\Delta_1+\frac{h^2}{2(u+\Delta_1)}+v+\Delta_2+\frac{h^2}{2(v+\Delta_2)}$$

$$\Rightarrow (\mu-1)(\Delta_1+\Delta_2) = \frac{h^2}{2}\left[\frac{1}{u+\Delta_1}+\frac{1}{v+\Delta_2}\right] \quad \text{——— (6)}$$

But $\Delta_1 << u$ and $\Delta_2 << v$ so $u+\Delta_1 \approx u$ and $v+\Delta_2 \approx v$

From eqn (6), $$(\mu-1)(\Delta_1+\Delta_2) = \frac{h^2}{2}\left(\frac{1}{u}+\frac{1}{v}\right) \quad \text{——— (7)}$$

From fig-2, we can write

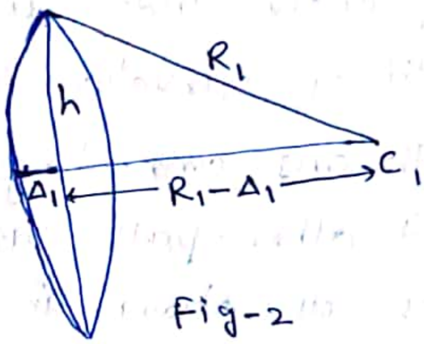

Fig-2

$$R_1-\Delta_1 = \sqrt{R_1^2-h^2}$$

$$\Rightarrow \Delta_1 = R_1-\sqrt{R_1^2-h^2}$$

$$= R_1-R_1\left[1-\frac{h^2}{R_1^2}\right]^{\frac{1}{2}}$$

$$\simeq R_1-R_1\left[1-\frac{1}{2}\frac{h^2}{R_1^2}\right] \quad \text{By using Binomial theorem}$$

$$= R_1-R_1+\frac{h^2}{2R_1}$$

$$\Rightarrow \Delta_1 = \frac{h^2}{2R_1} \quad \text{——— (8)}$$

Similarly $$\Delta_2 = \frac{h^2}{2R_2} \quad \text{——— (9)}$$

Using eqns (8) and (9) in eqn (7), we get

$$(\mu-1)\left(\frac{h^2}{2R_1}+\frac{h^2}{2R_2}\right) = \frac{h^2}{2}\left(\frac{1}{v}+\frac{1}{u}\right)$$

$$\Rightarrow \frac{1}{v}+\frac{1}{u} = (\mu-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \quad \text{——— (10)}$$

Using sign convention, $u=-u$, $v=+v$, $R_1=+R_1$ and $R_2=-R_2$ in eqn (10), we get

$$\boxed{\frac{1}{v}-\frac{1}{u} = (\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)} \quad \text{——— (11)}$$

This is the thin lens formula