(5) Packing fraction or Density of packing (F) ⇒.
The fraction of total vol. of the unit cell occupied by the atoms is called packing fraction. It may also be defined as the ratio of vol. occupied by the atoms in unit cell to the total vol. of the unit cell.

Packing fraction (F) = $\frac{\text{Vol. occupied by the atoms in unit cell}}{\text{Total vol. of the unit cell}}$

$$F = \frac{v}{V}$$

* Packing efficiency or % of packing = P.F × 100
(% of total space occupied by the particles in unit cell)

$$= \frac{v}{V} \times 100$$

* The fraction of vacant space can be calculated from the % packing fraction.

Ex (1) In SCC unit cell ⇒.

$v = z \times \text{vol. of one atom} = 1 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{a}{2}\right)^3 = \ldots$

$V = a^3$

$P.F = \frac{v}{V} = \frac{\frac{\pi}{6}a^3}{a^3} = \frac{\pi}{6} = \frac{3.14}{6} = 0.5236$

% of Packing fraction (or packing efficiency) = 100 × P.F
= 100 × 0.5236
= 52.36

% of vacant space = 100 − % of P.F = 100 − 52.36 = 47.64.

fraction of vacant space = 0.4764

i.e, about 52% of the SCC unit cell occupied by the atoms and ≈ 48% is empty or vacant.

II In FCC unit cell → $z = 4$

$$u = z \times \frac{4}{3}\pi r^3 = 4 \times \frac{4}{3}\pi\left(\frac{\sqrt{2}a}{4}\right)^3 = \frac{\sqrt{2}\pi a^3}{6} = \frac{\pi a^3}{3\sqrt{2}}$$

$$V = a^3$$

$$P.F = \frac{u}{V} = \frac{\pi a^3}{3\sqrt{2}} \Big/ a^3 = \frac{\pi}{3\sqrt{2}} = \frac{3.14}{3 \times 1.414} = 0.7406$$

% of P.F = 100 × PF = 100 × 0.7406 = 74.06 ≈ 74

% of vacant space = 100 − 74.06 = 25.94 ≈ 26

fraction of vacant space in unit cell = 1 − P.F = 1 − 0.740
= 0.25
≈ 0.

i.e. ≈ 74 % of the unit cell is occupied by the atoms and ≈ 26
empty.

III In BCC unit cell → $z = 2$

$$u = z \times \frac{4}{3}\pi r^3 = 2 \times \frac{4}{3}\pi\left(\frac{\sqrt{3}}{4}a\right)^3 = \frac{\sqrt{3}}{8}\pi a^3$$

$$V = a^3$$

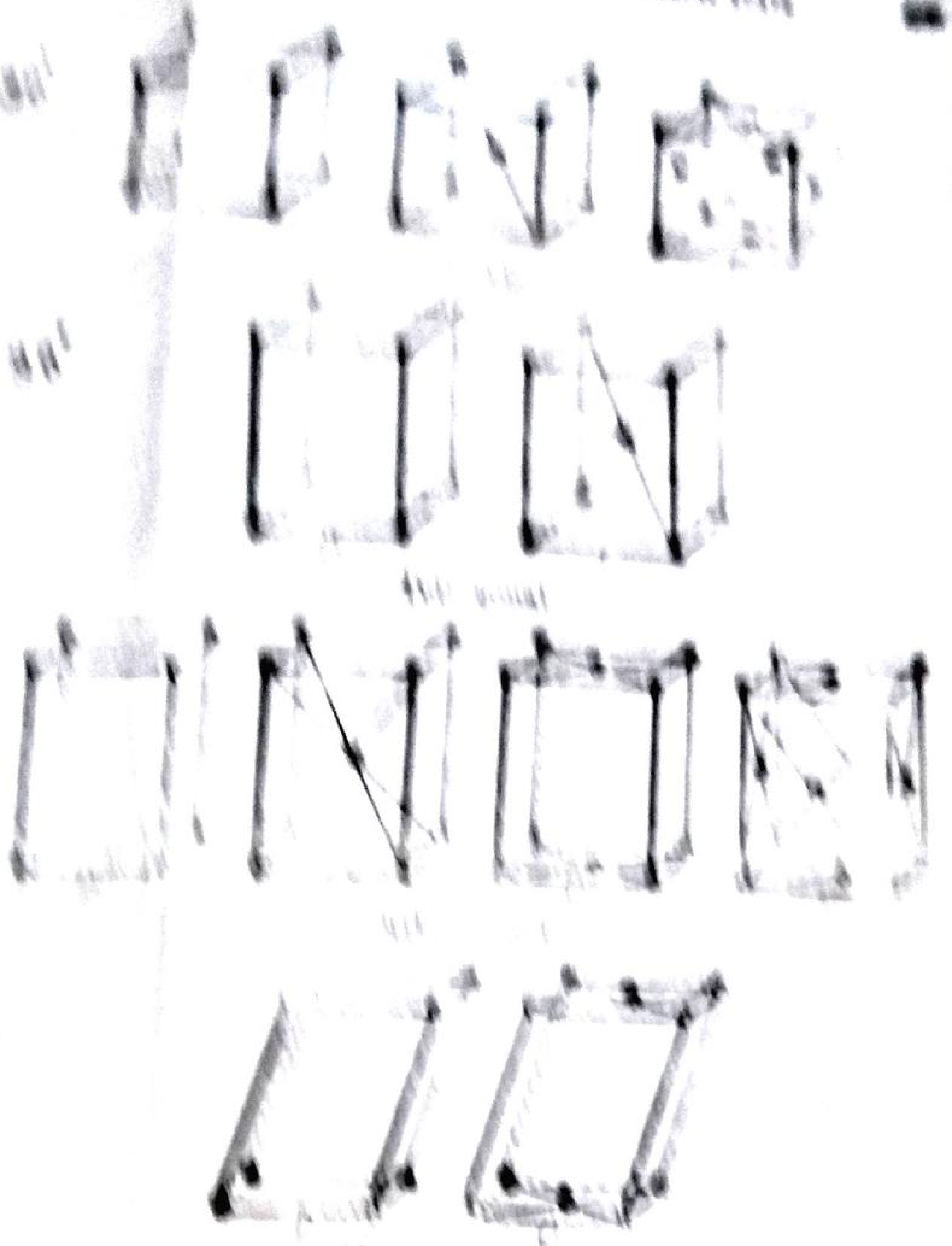
$$P.F = u/V = \frac{\sqrt{3}}{8}\pi a^3 \Big/ a^3 = \frac{\sqrt{3}\pi}{8} = \frac{1.732 \times 3.14}{8} = 0.6$$

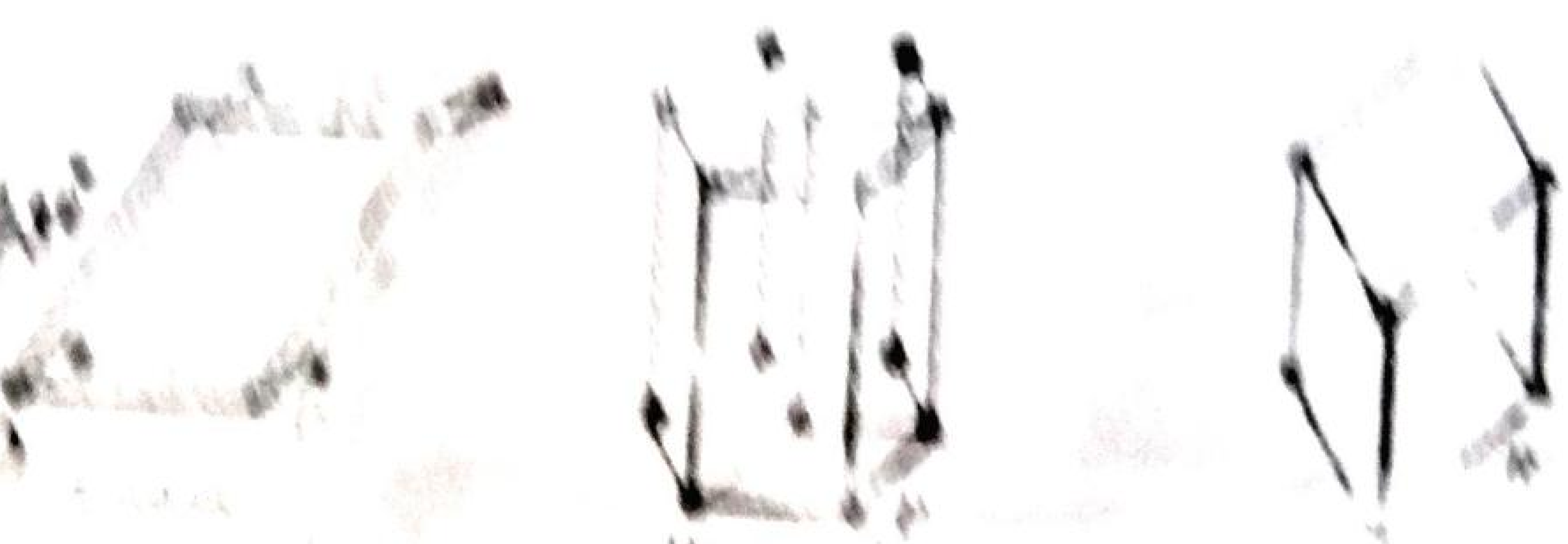
% of PF = 100 × 0.6802 = 68.02

% of vacant space = 100 − 68.02 = 31.98

fraction of vacant space = 1 − 0.6802 = 0.3198

i.e. in BCC unit cell ≈ 68 % space occupied by the atoms
≈ 32 % is empty.





$a = b \neq c$	$a = b = c$
$\alpha = \beta = 90^\circ$	$\alpha \neq \beta \neq \gamma \neq 90^\circ$
$\gamma = 120^\circ$	Quartz, $NaNO_3$