<u>TO FIND THE VALUE OF</u> $\frac{1}{f(D)} x^n \sin ax.$

Now $\quad \frac{1}{f(D)} x^n (\cos ax + i \sin ax) = \frac{1}{f(D)} x^n e^{iax} = e^{iax} \frac{1}{f(D+ia)} x^n$

$$\boxed{\frac{1}{f(D)} \cdot x^n \sin ax = \text{ Imaginary part of } e^{iax} \frac{1}{f(D+ia)} \cdot x^n}$$

$$\boxed{\frac{1}{f(D)} \cdot x^n \cos ax = \text{ Real part of } e^{iax} \frac{1}{f(D+ia)} \cdot x^n}$$

Q.12 Solve the differential equation :

$(D^2 + 2D + 1) y = x \cos x$

Sol. $(D^2 + 2D + 1) y = x \cos x$

Auxiliary equation is

$$m^2 + 2m + 1 = 0 \qquad \Rightarrow \qquad (m+1)^2 = 0 \qquad \Rightarrow \qquad m = -1, -1$$

$$\text{C.F.} = (C_1 + C_2 x)\ e^{-x}$$

$$\text{P.I.} = \frac{1}{(D+1)^2} x \cos x = \text{ Real part of } \frac{1}{(D+1)^2} x[\cos x + i \sin x]$$

$$= \text{Real part of } \frac{1}{(D+1)^2} x e^{ix} = \text{Real part of } e^{ix} = \frac{1}{(D+i+1)^2} x$$

$$= \text{Real part of } e^{ix} = \frac{1}{D^2 + 2(i+1)D + (i+1)^2} x = \text{Real part of } e^{ix} = \frac{1}{D^2 + 2(1+i)D + 2i} x$$

$$= \text{Real part of } \frac{e^{ix}}{2i} = \frac{1}{1 + \frac{1+i}{i} D + \frac{D^2}{2i}} x = \text{Real part of } \frac{e^{ix}}{2i} = \left[1 + \frac{1+i}{i} D + \frac{D^2}{2i}\right]^{-1} x$$

$$= \text{Real part of } \frac{e^{ix}}{2i} = \left[1 - \left(\frac{1+i}{i}\right) D + \ldots\right] x = \text{Real part of } \frac{1}{2i} (\cos x + i \sin x) \left[x - \frac{1+i}{i}\right]$$

$$= \text{Real part of } -\frac{1}{2} (i \cos x - \sin x)(x + i - 1)$$

$= \text{Real part of } -\frac{1}{2}(i\cos x - \sin x)(x + i - 1) = \frac{1}{2}\sin x(x-1) + \frac{1}{2}\cos x$

Complete solution is $y = C.F. + P.I.$

$\Rightarrow \quad y = (C_1 + C_2 x)e^{-x} + \frac{1}{2}(x-1)\sin x + \frac{1}{2}\cos x$

Q.13 Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x\sin x$

Sol. Auxiliary equation is $m^2 - 2m + 1 = 0 \quad \Rightarrow$

$C.F. = (C_1 + C_2 x)\, e^x$

P.I. $= \frac{1}{D^2 - 2D + 1} x \cdot \sin x \qquad (e^{ix} = \cos x + i\sin x)$

$= \text{Imaginary part of } \frac{1}{D^2 - 2D + 1} x(\cos x + i\sin x) = \text{Imaginary part of}$
$\frac{1}{D^2 - 2D + 1} x \cdot e^{ix}$

$= \text{ Imaginary part of } e^{ix} \frac{1}{(D+i)^2 - 2(D+i) + 1} \cdot x$

$= \text{ Imaginary part of } e^{ix} \frac{1}{D^2 - 2(1-i)D - 2i} \cdot x$

$= \text{ Imaginary part of } e^{ix} \frac{1}{-2i}\left[1 - (1+i)D - \frac{1}{2i}D^2\right]^{-1} \cdot x$

$= \text{ Imaginary part of } (\cos x + \sin x)\left(\frac{i}{2}\right)[1 + (1+i)D]x$

$= \text{ Imaginary part of } \frac{1}{2}(i\cos x - \sin x)[x + 1 + i]$

P.I. $= \frac{1}{2}x\cos x + \frac{1}{2}\cos x - \frac{1}{2}\sin x$

Complete solution is $y = (C_1 + C_2 x)e^x + \frac{1}{2}(x\cos x + \cos x - \sin x)$