BBA SEM-V **Operations Research**

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Transportation Problem

The transportation problem is a special type of linear programming problem where the 'objective is to minimise the cost of distributing a product from a number of sources or origins to a number of destinations. Because of its special structure the usual simplex method is not suitable for solving transportation problems. These problems require a special method of solution. The origin of a transportation problem is the location from which shipments are dispatched. The destination of a transportation problem is the location to which shipments are transported. The unit transportation cost is the cost of transporting one unit of the consignment from an origin to a destination.

In the most general form, a transportation problem has a number of origins and a number of destinations. A certain amount of a particular consignment is available in each origin. Likewise, each destination has a certain requirement. The transportation problem indicates the amount of consignment to be transported from various origins to different destinations so that the total transportation cost is minimised without violating the availability constraints and the requirement constraints. The decision variables *Xij* of a transportation problem indicate the amount to be transported from the *ith* origin to the *jth* destination. Two subscripts are necessary to describe these decision variables. A transportation problem can be formulated as a linear programming problem using decision variables with two subscripts.

Example: A manager has four Factories (i.e. origins) and four Warehouses (i.e. destinations). The quantities of goods available in each factory, the requirements of goods in each warehouse and the costs of transportation of a product from each factory to each warehouse are given. His objective is to ascertain the quantity to be transported from various factories to different warehouses in such a way that the total transportation cost is minimised.

Balanced Transportation Problem

Balanced Transportation Problem is a transportation problem where the total availability at the origins is equal to the total requirements at the destinations. For example, in case the total production of 4 factories is 1000 units and total requirements of 4 warehouses is also 1000 units, the transportation problem is said to be a balanced one.

Unbalanced Transportation Problem

Unbalanced transportation problem is a transportation problem where the total availability at the origins is not equal to the total requirements at the destinations. For example, in case the total production of 4 factories is 1000 units and total requirements of 4 warehouses is 900 units or 1,100 units, the transportation problem is said to be an unbalanced one. To make an unbalanced transportation problem, a balanced one, a dummy origins) or a dummy destination (s) (as the case may be) is introduced with zero transportation cost per unit.

Dummy Origin/Destination

A dummy origin or destination is an imaginary origin or destination with zero cost introduced to make an unbalanced transportation problem balanced. If the total supply is more than the total demand we introduce an additional column which will indicate the surplus supply with transportation cost zero. Likewise, if the total demand is more than the total supply, an additional row is introduced in the Table, which represents unsatisfied demand with transportation cost zero.

Practical Steps Involved In Solving Transportation Problems Of Minimization Type

The practical steps involved in solving transportation problems of minimization type are given below:

Step 1- See whether Total Requirements are equal to Total Availability; if yes, go to Step 2; if not, Introduce a Dummy Origin/Destination, as the case may be, to make the problem a balanced one Taking Transportation Cost per unit as zero for each Cell of Dummy Origin/Destination or as otherwise indicated.

Step 2- Find Initial Feasible Solution by following either the Least Cost Method(or LCM) or North-West Comer Method (or NWCM) or Vogel's Approximation Method (or VAM)

Step 3- After obtaining the Initial Feasible Solution Table, see whether Total Number of Allocations are equal to *"m + n* - 1",; if yes, go to Step 4; if not, introduce an infinitely small quantity Independent Cell. (i.e., for which no Loop can be formed).

Note: Introduce as many number of *'e'* as the total number of allocated cells falls below *"m + n* -1".

Step 4-

Optimality Test: Carry out the Optimality Test on the InitialSolution Table to find out the optimal solution.

Step 5 – Calculate the Total Minimum Cost = S (X ij x C ij),

where, X = Units Allocated to a Cell;

C = Shipping Cost per Unit of a Cell;

i = Row Number;

j = Column Number

Example : A concrete company transports concrete from three plants, 1, 2 and 3, to three construction sites, A, B and C. The plants are able to supply the following numbers of tons per week: Plant Supply (capacity) 1 300 2 300 3 100 .The requirements of the sites, in number of tons per week, are: Construction site Demand (requirement) A 200 B 200 C 300.The unit cost of Transpotation are given in the cells.

Transportation Table:

To ⟶ From ⇩	A	B	C	Supply
1	4	3	8	300
2	7	5	9	300
3	4	5	5	100
Demand	200	200	300	

Methods for Obtaining Basic Feasible Solution for Transportation Problem

The first step in using the transportation method is to obtain a feasible solution, namely, the one that satisfies the rim requirements (i.e. the requirements of demand and supply).

The initial feasible solution can be obtained by several methods. The commonly used methods are:

(I) North west Corner Method

(II) Least Cost Method (LCM)

(III) Vogal's Approximation Method (VAM)

Initial solution

(I) North West Corner Method :

The North West corner rule is a method for computing a basic feasible solution of a transportation problem where the basic variables are selected from the North – West corner (i.e., top left corner).

Steps:

1. Select the north west (upper left-hand) corner cell of the transportation table and allocate as many units as possible equal to the minimum between available supply and demand requirements, i.e., min (s1, d1).

2. Adjust the supply and demand numbers in the respective rows and columns allocation.

3. If the supply for the first row is exhausted then move down to the first cell in the second row.

4. If the demand for the first cell is satisfied then move horizontally to the next cell in the second column.

5. If for any cell supply equals demand then the next allocation can be made in cell either in the next row or column.

6. Continue the procedure until the total available quantity is fully allocated to the cells as required.

7. Check to make sure that the capacity and requirements are met.

Solution

To → From ↓	A	B	C	Supply
1	4 (200)	3(100)	8	~~300(100)~~
2	7	5(100)	9(200)	~~300(200)~~
3	4	5	5(100)	~~100~~
Demand	~~200~~	~~200(100)~~	~~300~~	

Total number of allocation=5

m+n -1=3+3-1 =5

m=no. of rows

n= no. of columns

Total cost=4*200+3*100+5*100+9*200+5*100= Rs.4300

Problem:Determine the initial basic feasible solution (IBFS) by North West Corner Method

Origin/Destination	D_1	D_2	D_3	D_4	Supply
S_1	21(6)	16(5)	15	3	11
S_2	17	18(5)	14(8)	23	13
S_3	32	27	27(4)	41(15)	19
Demand	6	10	12	15	

Calculation of Total Cost:

Total Cost= 21*6+16*5+18*5+14*8+27*4+41*15=Rs.1131

Assignment

Exercise 1.BBA SEM V (2016-19)

Source/Destination	D_1	D_2	D_3	D_4	Supply
S_1	6	2	3	9	10
S_2	4	5	5	1	15
S_3	7	3	4	8	4
S_4	5	1	2	7	6
Demand	6	10	15	4	35

Exercise 2. BBA SEM V(2016-2019)

Factory/Retail	R_1	R_2	R_3	R_4	Supply
F_1	21	16	25	13	11
F_2	17	18	14	23	13
F_3	32	27	18	41	19
Demand	6	10	12	15	43

Exercise 3. (2015-2018)

Factory/Warehouse	W_1	W_2	W_3	W_4	Supply
F_1	95	80	70	60	70
F_2	75	65	60	50	40
F_3	70	45	50	40	90
F_4	60	40	40	30	30
Demand	40	50	60	60	210/230

VAM Method

Exercise 4. (2015-2018)

Manufacturing centre/Depot	A	B	C	D	E	Supply
P	4	1	3	4	4	60
Q	2	3	2	2	3	35
R	3	5	2	5	4	40
Demand	22	45	20	18	30	135

Exercise 5.(2015-2018)

Factory/Warehouse	W_1	W_2	W_3	W_4	Supply
F_1	21	16	25	13	22
F_2	17	18	14	23	35
F_3	32	27	18	41	50
Demand	25	25	7	50	107

Vogal's Approximation Method

Steps:

1.Find the penalty cost by finding the difference between the lowest costs and the next lowest costs of each row and column.

2.Among the penalties find the maximum penalty cost from the table.If maximum penalty is found in more than one row or column,the choose any one of them.

3. Find out the cell in the selected row or column that has least cost.Allocate to this cell as much as possible depending on the supply and demand requirement.

4. Again compute the column and row penalties for reduced transportation table and go to repeat the step 2.

Exercise 1

Source/Destination	D_1	D_2	D_3	Supply
S_1	0	2	1	6
S_2	2	1	5	7
S_3	2	4	3	7
Demand	5	5	10	20